PhD research topic proposal
BME, Doctoral School of Mathematics and Computer Science

Name and degree of supervisor :

Gergely Kiss, PhD

Are you willing to supervise Stipendium Hungaricum applicants?

No

Title of the topic:

Fuglede's conjecture on Abelian groups

Short description:

Fuglede's conjecture on finite Abelian groups is an intensely studied topic in discrete harmonic analysis nowadays. The conjecture basically states that a set Ω tiles a group if and only if there exists an orthogonal basis of $L^2(\Omega)$.

Formally, for a locally compact Abelian group G and a Haar measure m defined on G we say that a set Ω of G with $0 \leq m(\Omega) \leq \infty$ is a tile, if there exists a set T in G such that every element of G can be uniquely written as the sum of elements of Ω and T up to a set of measure zero. We say that S is spectral if there is a set L in Irr(G) (i.e. the characters of G) such that the elements of L form an orthogonal basis in $L^2(\Omega)$. Fuglede's conjecture states that S is spectral if and only if S is a tile.

Thanks to T. Tao, M. Matolcsi, M. Kolountzakis, B. Farkas, P. Móra we know that the conjecture is false in $\mathbf{R^n}$ for n>2. On the other hand, it is still open for n=1 and n=2. For finite Abelian groups many examples and counterexamples are known for the conjecture, and still many open directions do exist. For instance: Does Fuglede conjecture holds for (a) cyclic groups of squarefree order; (b) cyclic groups of order $p^n q^m$, where p,q are odd primes and n,m are natural numbers; (c) $(\mathbf{Z}_p)^3$, where p is an odd prime; (d) $\mathbf{Z}_{p^n}$ x $\mathbf{Z}_{p^m}$, where n,m are natural numbers and p is a prime?

Requirements:

Analysis (Measure Theory, Fourier and Complex Analysis), Functional Analysis, Algebra (Representation Theory), Geometry (Finite and Projective Geometry)

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Place of work:

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