PhD thesis research topic
Doctoral School of Mathematics and Computer Science, Budapest Univ. Techn.

Name of supervisor, research degree (in case of external supervisor also the data of the departmental supervisor):
Supervisor: Gábor Domokos, member of of Hungarian Academy of Sciences
Co-supervisor: Zsolt Lángi, PhD

The title of the PhD topic:
Morphodynamics of convex solids

Brief description of the task:
The seminal proof of Perelman of the Poincaré conjecture relied on a set of geometric evolution equations, called curvature-driven flows, invented by Hamilton just for the purpose to serve this proof. As often before, it turned out only afterwards that these equations are not only the ideal tools for the purpose of pure mathematics but they also describe a broad class of natural abrasion phenomena which play a central role in geomorphology and planetology.

In this project we propose to explore curvature-driven flows with a focus on one particularly appealing, not-yet-fully-explored aspect of natural abrasion processes: the observed fact that the number of static equilibrium points decreases in this process. The monotonicity of this evolution appears to be of key geophysical significance, yet its mathematical description is lacking.

An equilibrium point of a convex solid is a stationary point of the distance function measured from the center of gravity, placing the solid on a horizontal plane it can be statically balanced at these points. We can distinguish between maximum, minimum and saddle points, the numbers of which we denote by S, U and H, respectively. In case of convex solids, the Poincaré-Hopf Theorem implies the relationship

$$S+U-H=2,$$

and based on this any convex solid can be assigned to an {S,U} equilibrium class. In addition to the number of equilibrium points, the topology of the integral curves in the gradient flow connecting these points also describes convex solids. Based on this aspect, within each equilibrium class we can distinguish topological subclasses. Equilibrium classes and subclasses offer a powerful tool to classify natural shapes. Our former research verified that both the system of equilibrium and that of topological classes are complete in the sense that there is neither empty class, nor empty subclass.

In the present PhD research, we investigate the evolution of shapes in these classification systems. Our goal, among others, is to find out how robust these classes and subclasses are; that is, by what probability a convex solid can move from one class or subclass into another one by abrasion. We can also track abrasion processes via computer models and by state of the art experimental equipment. Our goal is to compare these data to the mathematical models. We already have some initial results, but many questions are not yet answered which are essential from physical applications.

Expectations for the applicant (e.g. knowledge of foreign languages, deeper knowledge of certain areas of mathematics, etc.):
The topic essentially is geometrically motivated, within this knowledge of classical differential geometry is important. Expertise in low-dimensional dynamical systems is an asset, and also familiarity with numeric computations and programming is very useful. The topic has also statistical aspects, we welcome applicants with such interest as well. Primarily we expect the applications of students with a degree in mathematics or physics.

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Research place (name of the department, in case of external supervisor also the name of the external research place): Department of Geometry

Declaration
The conditions for research in the suggested topic are satisfactory at the department, the announcement of the topic has been approved by the department head.