PhD thesis research topic
Doctoral School of Mathematics and Computer Science, Budapest Univ. Techn.

Name of supervisor, research degree (in case of external supervisor also the data of the departmental supervisor):
Supervisor: Gábor Domokos, member of of Hungarian Academy of Sciences
Co-supervisor: Zsolt Lángi, PhD

The title of the PhD topic:
Convex mosaics

Brief description of the task:
Research on mosaics, i.e. families of regions filling space without gaps or overlaps, which was started possibly with Plato's theory of the universe composed of regular solids, has been in the focus of research ever since. It is easy to show that if all such regions are convex then they can be only politopes; we refer to this case as a *convex mosiac* and denote the number of vertices of a politope by $\underline{v}$ (vertex degree) and the number of politopes overlapping at one vertex by $\underline{n}$ (nodal degree). While convex mosaics are a thoroughly investigated area of convex geometry, still, the total number (28) of uniform convex mosaics (i.e. convex mosaics having identical nodes and having politopes with regular faces) has only been established recently, also, the most significant results on random convex mosaics only appeared a few decades ago.

Here we pick up Plato's idea and use mosaics as models for natural fragmentation of surfaces and solids. This approach leads to *mean field theories* where we study how global averages (belonging to either deterministic or random mosaics) may characterize the local geometry of a convex mosaic, in particluar, we are interested in the averages $\underline{v}$, $\underline{n}$ corresponding to $\underline{v}$ and $\underline{n}$. For 2-dimensional mosaics there exists a formula connecting these quantities, however, in higher dimensions no such formula is known. Still, when looking at the 28 uniform mosaics it becomes intuitively apparent that the averages $\underline{v}$, $\underline{n}$ are related. This intuition has been formalized in a conjecture stating that the *harmonic degree* $\underline{h}=\underline{n}\underline{v}/(\underline{n}+\underline{v})$ of any d-dimensional convex (and face-to-face) mosaic should be in the range $(d,2^{d-1}]$ for any $d > 2$. This conjecture (if true) implies major consequences not only for applications, but also for the geometric theory of mosaics.

The aim of this project is to study the combinatorial properties of mosaics using mean field theories, primarily (but not exclusively) convex, face-to-face mosaics. Some of the questions that may arise during research are the following.

- Prove (special cases of) the conjecture for the admissible interval of the harmonic degree for convex, face-to-face mosaics.
- Prove estimates or formulate conjectures for the non-convex, non-face-to-face scenarios.
- Prove upper bounds for $\underline{n}$ and $\underline{v}$ defined in the previous problem.
- Examine analogous quantities in case of packings and coverings of convex polytopes.

Expectations for the applicant (e.g. knowledge of foreign languages, deeper knowledge of certain areas of mathematics, etc.):
The topic esentially is geometrically motivated, within this knowledge of classical and convex geometry is important. Familiarity with numeric computations and programming is very useful. The topic has also statistical aspects, we welcome applicants with such interest as well. Primarily we expect the applications of students with a degree in mathematics or physics, but students with engineering degee and a wider mathematical background are also welcome.

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Research place (name of the department, in case of external supervisor also the name of the external research place): Department of Geometry

Declaration

The conditions for research in the suggested topic are satisfactory at the department, the announcement of the topic has been approved by the department head.