PhD research topic proposal
BME, Doctoral School of Mathematics and Computer Science

Name of supervisor :

Dr. Szirmai Jenő

Degree:

PhD, habil.

Title of the topic:

Ball packings, coverings and Dirichlet-Voronoi cells in Thurston geometries

Short description:

The classical sphere packing problems concern arrangements of non-overlapping equal spheres (rather balls) which fill a space. Space is the usual three-dimensional Euclidean space. However, ball (sphere) packing problems can be generalized to the other 3-dimensional Thurston geometries

$$\mathbf{E}^3,\ \mathbf{S}^3,\ \mathbf{H}^3,\ \mathbf{S}^2\times\mathbf{R},\ \mathbf{H}^2\times\mathbf{R},\ \widetilde{\mathbf{SL}_2\mathbf{R}},\ \mathbf{Nil},\ \mathbf{Sol}$$

and to higher dimensional various spaces.

In an n-dimensional space of constant curvature d_n(r) be the density of n+1 spheres of radius r mutually touching one another with respect to the simplex spanned by the centres of the spheres. L. Fejes Tóth and H.S.M. Coxeter conjectured that in an n-dimensional space of constant curvature the density of packing spheres of radius r cannot exceed dn(r).
This conjecture has been proved by C. Roger in the Euclidean space. The 2-dimensional case has been solved by L. Fejes Tóth. In an 3-dimensional space of constant curvature the problem has been investigated by Böröczky and Florian and it has been studied by K. Böröczky for n-dimensional space of constant curvature (n> 3).

We have studied some new aspects of the horoball and hyperball packings in n-dimensional hyperbolic space and we have realized that the ball, horoball and hyperball packing problems are not settled yet in the n-dimensional n>2 hyperbolic space.

The goal of this PhD program to generalize the above problem of finding the densest geodesic and translation ball (or sphere) packing and covering to the other 3-dimensional homogeneous geometries (Thurston geometries) $\mathbf{S}^2\times\mathbf{R}$, $\mathbf{H}^2\times\mathbf{R}$, $\widetilde{\mathbf{SL}_2\mathbf{R}}$, $\mathbf{Nil}$, $\mathbf{Sol}$.
Moreover, we will study the structures of Dirichlet-Voronoi cells related to the packing configurations and the lattices in Sol and Nil geometries.

We note here that the greatest known packing density is realized in $\mathbf{S}^2\times\mathbf{R}$ geometry with packing density is ~0.87499429 .

We will use the unified interpretation of the Thurston geometries in the projective 3-sphere.

Further informations: www.math.bme.hu/~szirmai

Requirements:

MSc degree in mathematics, strong background in geometry.

Contact:

[redacted]

Place of work:

BME, MI, Department of Geometry.

Statement*:*

The conditions of the research above are satisfied, the theme is confirmed by the Head of the Department