PhD thesis research topic
Doctoral School of Mathematics and Computer Science, Budapest Univ. Techn.

Name of supervisor, research degree (in case of external supervisor also the data of the departmental supervisor):
Supervisor: Gábor Domokos, member of of Hungarian Academy of Sciences
Co-supervisor: Zsolt Lángi, PhD

The title of the PhD topic:
Morphodynamics of convex solids

Brief description of the task:
A physically extremely interesting area of convex geometry deals with the description of convex solids with the aim to identify, categorize and track the evolution of natural shapes. In addition to convex geometry, mathematical tools include geometric partial differential equations, in particular, curvature-driven flows which are closely related to the heat equation. An equilibrium point of a convex solid is a stationary point of the distance function measured from the center of gravity, placing the solid on a horizontal plane it can be statically balanced at these points. We can distinguish between maximum, minimum and saddle points, the numbers of which we denote by S, U and H, respectively. In case of convex solids, the Poincaré-Hopf Theorem implies the relationship

$$S+U-H=2,$$

and based on this any convex solid can be assigned to an {S,U} equilibrium class. In addition to the number of equilibrium points, the topology of the integral curves in the gradient flow connecting these points also describes convex solids. Based on this aspect, within each equilibrium class we can distinguish topological subclasses. By the equilibrium class and subclasses, a very interesting and from geological point of view very useful classification system can be defined for shapes that can be found in nature (e.g. pebble shapes). Our former research verified that both the system of equilibrium and that of topological classes are complete in the sense there is neither empty class, nor empty subclass.

In the present PhD research we investigate some particularly interesting geometric properties of the above defined classification system, and also other possible classification systems. Our goal, among other things, is to find out how robust these classes and subclasses are; that is, by what probability a convex solid can move from one class or subclass into another one by abrasion. Our aim is a deeper understanding of the partial differential equations describing abrasion processes, and proving statements related to them. We can also track abrasion processes via computer models and by state of the art experimental equipment. Our goal is to compare these data to the mathematical models. We already have some inital results, but many questions are not yet answered which are essential from physical applications.

Expectations for the applicant (e.g. knowledge of foreign languages, deeper knowledge of certain areas of mathematics, etc.):
The topic esentially is geometrically motivated, within this knowledge of classical differential geometry is important. Expertise in low-dimensional dynamical systems is an asset, and also familiarity with numeric computations and programming is very useful. The topic has also statistical aspects, we welcome applicants with such interest as well. Primarily we expect the applications of students with a degree in mathematics or physics.

Contact information of the supervisor (in case of external supervisor also the data of the departmental supervisor):

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Contact information for the co-supervisor:
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Research place (name of the department, in case of external supervisor also the name of the external research place): Department of Geometry

Declaration

The conditions for research in the suggested topic are satisfactory at the department, the announcement of the topic has been approved by the department head.