

PhD research topic proposal (template)

BME, Doctoral School of Mathematics and Computer Science

Name and degree of supervisors :

Dr. Szirmai Jenő, PhD, habil

Are you willing to supervise Stipendium Hungaricum applicants?

No

Title of the topic:

Ball packings, coverings and Dirichlet-Voronoi cells in Thurston geometries

Short description:

The classical sphere packing problems concern arrangements of non-overlapping equal spheres (rather balls) which fill a space. Space is the usual three-dimensional Euclidean space. However, ball (sphere) packing problems can be generalized to the other 3-dimensional Thurston geometries

$$E^3, S^3, H^3, S^3 \times R, H^3 \times R, \tilde{SL}_2R, Nil, Sol$$

and to higher dimensional various spaces.

In an n -dimensional space of constant curvature $d_n(r)$ be the density of $n+1$ spheres of radius r mutually touching one another with respect to the simplex spanned by the centres of the spheres. L. Fejes Tóth and H.S.M. Coxeter conjectured that in an n -dimensional space of constant curvature the density of packing spheres of radius r cannot exceed $d_n(r)$.

This conjecture has been proved by C. Roger in the Euclidean space. The 2-dimensional case has been solved by L. Fejes Tóth. In an 3-dimensional space of constant curvature the problem has been investigated by Böröczky and Florian and it has been studied by K. Böröczky for n -dimensional space of constant curvature ($n > 3$).

We have studied some new aspects of the horoball and hyperball packings in n -dimensional hyperbolic space and we have realized that the ball, horoball and hyperball packing problems are not settled yet in the n -dimensional $n > 2$ hyperbolic space.

The goal of this PhD program to generalize the above problem of finding the densest geodesic and translation ball (or sphere) packing and covering to the other 3-dimensional homogeneous geometries (Thurston geometries) .

Moreover, we will study the structures of Dirichlet-Voronoi cells related to the packing configurations and the lattices in Sol and Nil geometries.

We note here that the greatest known packing density is realized in $S^2 \times R$ geometry with packing density is ~ 0.87499429 .

We will use the unified interpretation of the Thurston geometries in the projective 3-sphere.

Requirements:

MSc degree in mathematics, strong background in geometry.

Contact:

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Place of work:

BME, MI, Department of Algebra and Geometry.

