

PhD entrance exam problems – SAMPLE

At each statement please indicate whether it is true or false. A correct answer is worth 1 point, an incorrect one is -1 point, and an unanswered question is 0 point. You don't need to solve all problems, just solve as many as you can.

Analysis:

- (1) The sequence $1 + \frac{1}{n}$ converges to 1, and this implies that $(1 + \frac{1}{n})^n$ also converges to 1.
- (2) Let $f(x, y)$ be a continuous function on the unit square $[0, 1] \times [0, 1]$. Then $\int_{x=0}^1 \int_{y=0}^x f(x, y) dy dx = \int_{y=0}^1 \int_{x=y}^1 f(x, y) dx dy$.

Algebra, linear algebra:

- (3) Let A, B be real matrices of size $n \times n$. Then, for the trace and determinant of A and B we have $Tr(A + B) = TrA + TrB$ and $\det(A + B) = \det A + \det B$.
- (4) Let $p(x)$ be a polynomial of degree n with real coefficients, and $\alpha \in \mathbb{R}$, such that $p(\alpha) = 0$. Then $p(x)$ can be factorized as $p(x) = (x - \alpha)g(x)$ where $g(x)$ is a polynomial of degree $n - 1$ with real coefficients.

Probability:

- (5) A soccer player scores 0 goals in 60% of his games, 1 goal in 30%, and 2 goals in 10% (he never scores more than two goals). The expected number of his goals is 0.5 per game.
- (6) Let X be a real random variable with continuous density function $f(x)$. The probability of $a \leq X \leq b$ is given by $\int_a^b x f(x) dx$.

Discrete mathematics:

- (7) A basketball team consists of 5 players. A coach wants to select his team from a total of 8 candidates. He can make his selection in 28 different ways.
- (8) Any bipartite graph is triangle-free.

Geometry:

- (9) The equation $(x - 4)^2 + (y + 2)^2 = 9$ describes a circle of radius 3 in the (x, y) -plane.

- (10) Let $P \subset \mathbb{R}^3$ be a plane and $L \subset \mathbb{R}^3$ a line. Assume that the angle between P and L is 60° . Let $L' \subset P$ be a line in the plane P which intersects L . This implies that the angle between L and L' is automatically 60° .